

Exhibit A

Existence

157

Experiment

excess of nines. The remainder of a positive integer is divided by 9 when the greatest possible number of nines have been subtracted from it. The remainder determined by dividing a number by 9. It is customary to restrict the process to positive integers. The remainder of 237 is 3 (or $2 + 3 + 7 = 9 + 3$). The remainder of 1234 is 1 (or $1 + 2 + 3 + 4 = 10 + 3$). See SPHERICAL — SPHERICAL.

n. Payment of obligations by use of money; by use of orders, exchange of accounts, etc. Exchange carried on (between countries). The rate is the value of the money of one country in terms of the money of another (or versa).

n. excircle of a triangle. A circle tangent to one side of a triangle and to the extensions of the other two sides.

adj. law of the excluded middle. A law of contradiction.

adj. mutually exclusive. Two events which cannot occur at the same time.

n. method of exhaustion. A method invented by Eudoxus and Archimedes and Eudoxus) for finding the areas of circles, ellipses, and segments of circles. It consists of finding an increasing sequence of sets whose area is less than (greater than) the area of the set being approximated; then the area of the set approaches the area of the set being approximated as the area of the approximating sets approaches the area of the set being approximated.

adj. existence theorem. A theorem which asserts the existence of at least one object of a specified type. Example: The existence theorem of algebra states that any polynomial p of degree n with complex coefficients, there exists a complex number z for which $p(z) = 0$. It is a solution for a system of n equations in n unknowns if the determinant of the coefficients is nonzero; (3) If the function is continuous on the closed interval $[a, b]$ and y_1 are two real numbers between $y(a)$ and $y(b)$, then there is a value c such that $y(c) = y_1$. The solution y of the differential equation $y' = f(x, y)$ with initial condition $y(a) = y_1$ is unique.

$y' + f(x)y' + g(x)y = h(x)$ for which y'' is continuous on $[a, b]$, $y(a) = y_0$, and $y'(a) = y_1$. The argument that establishes an existence theorem is an *existence proof*. See UNIQUENESS — UNIQUENESS theorem.

EXISTENTIAL, adj. existential quantifier. A quantifier.

EXOTIC, adj. exotic four-space. See DONALD — DONALD.

EXOTIC sphere. Any manifold that has a differential structure of class C^∞ and is homeomorphic, but not diffeomorphic, to a natural sphere. See MANIFOLD.

EXPANDED, adj. expanded notation. The number represented by 537.2 in decimal notation can also be represented as $(5 \cdot 10^2) + (3 \cdot 10) + (7 \cdot 1) + (2 \cdot \frac{1}{10})$, which is an *expanded numeral*. An example of the use of *expanded notation*. Writing numerals with positional value understood, as for 537.2, is using *positional notation*.

EXPANSION, n. (1) The form a quantity takes when written as a sum of terms, or as a continued product, or in general in any type of *expanded (extended) form*. See FOURIER — FOURIER series, TAYLOR — Taylor's theorem. (2) The act, process, of obtaining the *expanded form* of a quantity. (3) Increase in size.

binomial expansion. The expansion given by the binomial theorem. See BINOMIAL — binomial theorem.

coefficient of linear expansion, thermal expansion and volume (or cubical) expansion. See COEFFICIENT.

expansion of a determinant. See DETERMINANT — expansion of a determinant.

expansion (of a function) in a series. Writing a function which converges to the function for given values of the variables (or which "represents" the function in some other sense). The function itself is also spoken of as the *expansion* of the function.

EXPECTATION, n. expectation of life. The average number of years that members of a given group may be expected to live after attaining a given age, according to a mortality table. Also, the complete expectation of life as distinguished from curtate expectation of life, which is the average number of *entire years* that members of a given group may be expected to live. Joint expectation of life is the average number of years that two (or more) persons at a given age may be expected to live, according to a mortality table.

mathematical expectation. Same as EXPECTED VALUE. See below.

EXPECTED, adj. expected value. For a discrete random variable X with values $\{x_n\}$ and probability function p , the expected value of X is $\sum x_n p(x_n)$, if this sum is finite or converges absolutely. For example, if two coins are tossed and Paul receives \$5 if they both show heads, \$1 if one shows heads and one shows tails, and pays \$6 if both show tails, then Paul's expectation is $5 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + (-6) \cdot \frac{1}{4} = \frac{1}{4}$ dollars. Most generally, if p is a probability measure defined on a set S , then the expected value of a function f is $\int f dp$ [see INTEGRABLE — integrable function]. In particular, if f is the probability density function (relative frequency function) of the random variable X , then

$$E(X) = \bar{X} = \int_{-\infty}^{+\infty} t f(t) dt$$

is the expected value of X . The quantity

$$\sigma^2 = \int_{-\infty}^{+\infty} (t - \bar{X})^2 f(t) dt$$

is the variance of X , or the expected value of the square of the deviation of X from its expected value (or mean). The expected value of the product XY of two independent variables X and Y is equal to the product of their expected values. If ϕ is a function of the random variable X whose probability density function is f , then

$$E[\phi(X)] = \int_{-\infty}^{+\infty} \phi(t) f(t) dt$$

is the expected value of ϕ . Syn. (arithmetic) average, expectation, mathematical expectation, (arithmetic) mean. See MEAN, MOMENT — moment of a distribution, SAMPLE — sample moment.

EXPENSES, n. overhead expenses. Administrative expenses, such as salaries of officers and employees, cost of supplies, losses by credit, and rent and plant depreciation. (It sometimes includes some of the selling expenses.)

selling expenses. Expenses such as insurance, taxes, advertising and salesmen's wages.

EXPERIMENT, n. (Statistics) Any operation or process of doing or observing something happen under certain conditions, resulting in some final "outcome." The collection of all possible

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